18. Prove that a finite Cartesian product of connected space is connected.

19. Prove that a subspace A of R^n is compact if and only if it is closed and is bounded in the Euclidean metric d or the square metric ρ.

20. State and prove Urysohn lemma.

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APRIL/MAY 2023

DMA32 — TOPOLOGY

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define a Topology on X.

2. Define the product topology.

3. Define the metric space.

4. If $d: X \times X \to R$ and defined by $d(x, y) = |x - y|$ check whether it is metric or not?

5. Define the linear continuum.

6. Define locally path connected.

7. Define compact.

8. Define uniform continuous function.

9. Define the dense.

10. Define completely regular.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) If X be a set and $\mathfrak{B}$ be a basis for topology τ on X, prove that τ equals the collection of all unions of elements of $\mathfrak{B}$.

Or

(b) Let A be a subset of the topological space X, let A' be the set of all limit points of A. Prove that $A = A \cup A'$.

12. (a) State and prove Pasting Lemma.

Or

(b) Let $f : X \to Y$; let X and Y be metrizable with metrics d_x and d_y respectively, then prove that the continuity of f is equivalent to the requirement that given $x \in X$ and given $\in > 0$, there exists $\delta > 0$ such that $d_x(x, y) \Rightarrow d_y(f(x), f(y)) < \in$.

13. (a) Prove that the union of a collection of connected subspaces of X that have a point in common is connected.

Or

(b) State and prove intermediate value theorem.

14. (a) Prove that every compact subspace of a Hausdorff space is closed.

Or

(b) State and prove uniform continuity theorem.

15. (a) Prove that a subspace of a regular space is regular, a product of regular spaces is regular.

Or

(b) Prove that every topological group is completely regular.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Let A be a subset of the topological space X. Prove that

(a) $x \in \overline{A}$ if and only if every open set U containing x intersects A.

(b) Supposing the topology of X is given by a basis, then $x \in \overline{A}$ if and only if every basis element B containing x intersects A.

17. Prove that the topologies on R^n induced by the Euclidean metric d and square metric ρ are the same as the product topology on R^n.

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